Population Growth and Technological Change: One Million B.C. to 1990
Author(s): Michael Kremer
Source: *The Quarterly Journal of Economics*, Vol. 108, No. 3 (Aug., 1993), pp. 681-716
Published by: Oxford University Press
Stable URL: https://www.jstor.org/stable/2118405
Accessed: 25-10-2019 20:04 UTC

POPULATION GROWTH AND TECHNOLOGICAL CHANGE: ONE MILLION B.C. TO 1990*

MICHAEL KREMER

The nonrivalry of technology, as modeled in the endogenous growth literature, implies that high population spurs technological change. This paper constructs and empirically tests a model of long-run world population growth combining this implication with the Malthusian assumption that technology limits population. The model predicts that over most of history, the growth rate of population will be proportional to its level. Empirical tests support this prediction and show that historically, among societies with no possibility for technological contact, those with larger initial populations have had faster technological change and population growth.

Models of endogenous technological change, such as Aghion and Howitt [1992] and Grossman and Helpman [1991], typically imply that high population spurs technological change. This implication flows naturally from the nonrivalry of technology. As Arrow [1962] and Romer [1990] point out, the cost of inventing a new technology is independent of the number of people who use it. Thus, holding constant the share of resources devoted to research, an increase in population leads to an increase in technological change. However, despite its ubiquity in the theoretical literature on growth, this implication is typically dismissed as empirically undesirable.

This paper argues that the long-run history of population growth and technological change is consistent with the population implications of models of endogenous technological change. The first section of the paper constructs a highly stylized model in which each person's chance of being lucky or smart enough to invent something is independent of population, all else equal, so that the growth rate of technology is proportional to total population. The model also makes the Malthusian [1978] assumption that population is limited by the available technology, so that the growth rate of population is proportional to the growth rate of

*I am grateful to Gene Grossman, Charles Jones, Gregory Mankiw, Paul Romer, Xavier Sala-i-Martin, James Thomson, many former classmates, an anonymous referee, and especially Robert Barro and Elhanan Helpman for assistance with this paper. Participants in seminars at Brown University, University of Chicago, Harvard University, and Yale University, and at the AEA meetings and the NBER Economic Growth and Economic Fluctuations Meetings provided useful comments. Jill Woodworth provided capable research assistance. I was supported by a National Science Foundation graduate fellowship while writing this paper.

The Quarterly Journal of Economics, August 1993

technology. Combining these assumptions implies that the growth rate of population is proportional to the level of population.

Figure I plots the growth rate of population against its level from prehistoric times to the present. The prediction that the population growth rate will be proportional to the level of population is broadly consistent with the data, at least until recently, when population growth rates have leveled off. The data, which are listed in Table I and discussed in Section IV, are drawn from McEvedy and Jones [1978], Deevey [1960], and the United Nations [various years]. While they are obviously subject to measurement error, there can be little doubt that the growth rate of population has increased over human history. Assuming that population has historically been limited by the level of technology, this much faster than exponential population growth is inconsistent with growth models which either assume constant exogenous technological change or generate it endogenously.

The model outlined in Section I is similar to that of Lee [1988], who combines the Malthusian and Boserupian interpretations of population history to generate accelerating growth of population. Lee adopts Boserup's [1965] argument that people are forced to adopt new technology when population grows too high to be supported by existing technology. However, this view is difficult to reconcile with the simultaneous rise in income and rates of

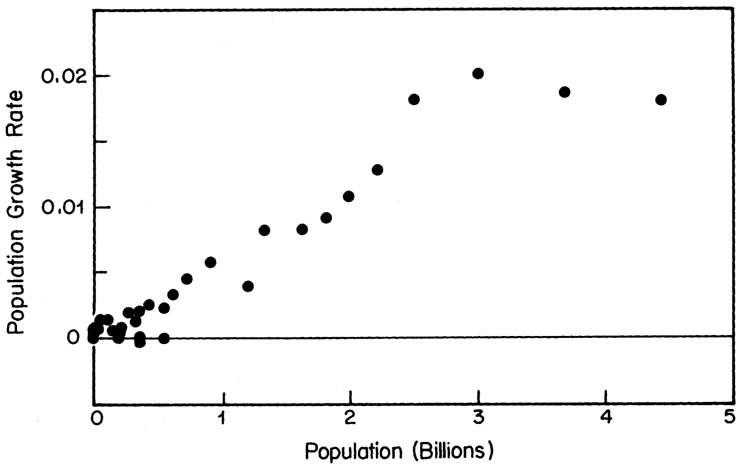

FIGURE I
Population Growth Versus Population

TABLE I
POPULATION GROWTH: 1,000,000 B.C. TO 1990

Year	Pop. (millions)	Growth rate	Comments
−1,000,000	0.125	0.00000297	
−300,000	1	0.00000439	
−25,000	3.34	0.000031	
−10,000	4	0.000045	
−5000	5	0.000336	
−4000	7	0.000693	
−3000	14	0.000657	
−2000	27	0.000616	
−1000	50	0.001386	
−500	100	0.001352	
−200	150	0.000623	
1	170	0.000559	
200	190	0.0	
400	190	0.000256	
600	200	0.000477	
800	220	0.000931	
1000	265	0.001886	
1100	320	0.001178	
1200	360	0.0	Mongol Invasions
1300	360	−0.0002817	Black Death
1400	350	0.0019420	
1500	425	0.002487	
1600	545	0.0	30 years war, Ming Collapse
1650	545	0.002253	
1700	610	0.003316	
1750	720	0.004463	
1800	900	0.005754	
1850	1200	0.003964	
1875	1325	0.008164	
1900	1625	0.008306	
1920	1813	0.009164	
1930	1987	0.010772	
1940	2213	0.012832	
1950	2516	0.018226	
1960	3019	0.020151	
1970	3693	0.018646	
1980	4450	0.018101	
1990	5333	—	

The growth rate listed for period t is the average growth rate from t to $t + 1$. Since differences of a constant at all times between different data sets would distort growth rates, the 25,000 to 10,000 B.C. growth rate is based on Deevey's population estimates, although the population estimate for 10,000 B.C. is from McEvedy and Jones. Similarly, the 1900–1920 growth rate is based on the 1900–1925 average annual growth rate from McEvedy and Jones. Population figures from 1920 to 1940 and from 1950 to 1980 are from the 1952 and 1985/6 editions of the *United Nations Statistical Yearbook,* respectively. The 1990 population estimate is from the 1991 *World Almanac* [1991], which attributes it to the U. S. Bureau of the Census.

technological change over most of history, since it implies that increases in income should have led to reduced effort to invent new technologies. In contrast, this paper argues that even if each person's research productivity is independent of population, total research output will increase with population due to the nonrivalry of technology. As Kuznets [1960] and Simon [1977, 1981] argue, a higher population means more potential inventors. Lee's model and the simple model of Section I each make different functional form assumptions about the effect of population on technological change and of technology on population. While these restrictive assumptions make the models tractable, they limit their ability to match certain features of the data, such as the recent decline in population growth rates.

Sections II and III generalize the simple model's assumptions about the determinants of research output and population, and show that for appropriate parameter values this generalized model is consistent with recent, as well as long-run, history. Section II generalizes the model to allow research productivity to increase with income, as seems appropriate in light of low research productivity in some densely populated countries, such as China. It shows that this can generate a negative cross-section relationship between population and research output, but leaves the time series implications of the model intact. Following Jones [1992], Section II further generalizes the model to allow research productivity to depend on population and the existing level of technology and shows that this generalized model can only be reconciled with the data if total technological change increases with population. An alternative model of exogenously increasing growth rates of technology, independent of population, is inconsistent with modern data.

Section III shows that if population grows at finite speed when income is above its steady state, rather than adjusting instantaneously, as in the simple model, per capita income will rise over time. If population growth declines in income at high levels of income, as is consistent with a variety of theoretical models and with the empirical evidence, this gradual increase in income will eventually lead to a decline in population growth.

Section IV empirically tests the model. Following Von Foerster, Mora, and Amiot [1960], subsection IV.A shows that as the model predicts, the growth rate of population has been proportional to its level over most of history. Subsection IV.B confirms the cross-section implications of the model by showing that among technologically separate societies, those with higher initial population had faster growth rates of technology and population. A conclusion summarizes the argument and discusses implications for policy and for the endogenous growth literature.

I. The Integrated Model: A Simple Version

This section quickly sketches a simple model of population growth and technological change along lines similar to those of Lee [1988]. It makes highly simplified assumptions about how technology affects population and how population affects the growth rate of technology, shows how they interact, and argues that a model combining these assumptions describes the data surprisingly well.

Assume that output is given by

$$Y = Ap^{\alpha}T^{1-\alpha}, \tag{1}$$

where A is the level of technology, p is population, and T is land, which is henceforth normalized to one.[1] Per capita income y therefore equals $Ap^{\alpha-1}$.

I assume that population increases above some steady state equilibrium level of per capita income, $\bar{y}$, and decreases below it. Diminishing returns to labor imply that a unique level of population, $\bar{p}$, generates income of $\bar{y}$:

$$\bar{p} = \left(\frac{\bar{y}}{A}\right)^{1/(\alpha-1)}. \tag{2}$$

In this simplified model I assume that population adjusts instantaneously to $\bar{p}$. Section III makes the more realistic assumption that population adjusts to $\bar{p}$ at finite speed. Note that increases in A, such as the invention of agriculture, shift the production function outward and raise the steady state population, $\bar{p}$.

Together with this Malthusian assumption about the determination of population by technology, the model adopts Kuznets' [1960] and Simon's [1977, 1981] view that high population spurs technological change because it increases the number of potential inventors. In particular, this simple model assumes that, all else equal, each person's chance of inventing something is independent of population. Thus, in a larger population there will be proportionally more people lucky or smart enough to come up with new

1. Allowing capital to enter the production function and setting the marginal product of capital equal to the discount rate does not substantially affect the analysis.

ideas.[2] If research productivity per person is independent of population and if A affects research output the same way it affects output of goods (linearly, by definition), then the growth rate of technology will be

$$\dot{A}/A = pg, \tag{3}$$

where g represents research productivity per person. Section II discusses a more general research equation.

Note that as long as technology can diffuse between countries, even with an arbitrarily long lag, equation (3) does not imply that countries with higher population will have faster technological change or economic growth. Belgium, for example, is rich not because it has invented a lot of technology, but because it has the human capital and social institutions that allow it to employ technology invented in other countries. Hence although Belgium has fewer people than Zaire, it has access to technologies invented by at least as many people. (Section IV shows that historically, among regions with no possibility for technological contact, those with higher populations had faster technological change.)

Combining the research and population determination equations is straightforward. Since population is limited by technology, the growth rate of population is proportional to the growth rate of technology. Since the growth rate of technology is proportional to the level of population, the growth rate of population must also be proportional to the level of population. To see this more formally, take the logarithm of the population determination equation, (2), and differentiate with respect to time:

$$\frac{\dot{p}}{p} = \frac{1}{1-\alpha}\frac{\dot{A}}{A}. \tag{4}$$

Substitute in the expression for the growth rate of technology, from (3), to obtain

$$\frac{\dot{p}}{p} = \frac{g}{1-\alpha}p. \tag{5}$$

This prediction, that the growth rate of population will be proportional to the level of population, implies much faster than exponential growth. In contrast, if there were a constant exoge-

2. Ted Baxter of the "Mary Tyler Moore Show" apparently agreed: he planned to have six children in the hope that one would solve the world's population problem.

nous growth rate of technology, or an endogenous growth rate independent of population, there would be no relationship between the level of population and its growth rate, and population would grow exponentially. Similarly, biological models of animal populations unconstrained by food supplies imply exponential growth. In biological models of constrained animal populations, the growth rate declines with population, as in the logistic pattern, $\dot{p}/p = 1 - p$, rather than increasing with population, as this model implies.

A first look at the data provided by Figure I indicates that this simple model matches the pattern of population growth over most of history. However, because of its restrictive assumptions, it does not match the recent leveling off and decline of population growth rates. The next two sections show that for appropriate parameter values, a generalized model is consistent with recent, as well as long-run, history.

II. The Effect of Population on Technological Change

This section generalizes the research equation of Section I to allow research productivity to depend on income, on the level of technology, and on population. It shows that if research productivity increases with income, the cross-section relationship between population and technological change is ambiguous, but that this does not alter the model's implication that technological change will increase as population grows over time. This section also shows that a general research equation proposed by Jones [1992], in which research productivity depends both on population and on the level of technology, is consistent with the history of population growth and technological change only if total research output increases at least proportionally with population. An alternative model, in which the growth rate of technology is independent of population and increases with the level of technology, is inconsistent with modern data.

A. Research Productivity as a Function of Income

Low research productivity in some poor, populous countries, such as India and China, suggests that research productivity may increase with income. As others, such as Young [1990], have argued, high population can reduce per capita income, and if research productivity is sensitive enough to income, this can reduce total research output. Thus, the cross-section relationship between population and technological change is ambiguous, as is the effect of exogenous policy-induced increases in population on technological change. I argue below, however, that this does not alter the time series relationship between population and technology outlined in Section I.

Assume that g, research productivity, equals ky^{δ}, where k and δ are positive parameters. Holding A constant and letting population vary due to temporary exogenous shocks, such as war, disease, or changes in tastes for children, the growth rate of technology will be proportional to $y^{\delta}p$, and since $y = Ap^{\alpha-1}$, to $p^{1+(\alpha-1)\delta}$. Hence total technological change increases with population if $\delta < 1/(1 - \alpha)$ and decreases with population if $\delta > 1/(1 - \alpha)$. A generous estimate of $1 - \alpha$, the share of land, might be about one-third, since the landlord's share in sharecropping contracts is usually less than one-half, and even extremely poor economies have nonagricultural activities to provide for food processing, clothing, and shelter. In this case, technological change would decrease in response to an exogenous increase in population only if each person's chance of inventing something increased faster than the cube of income. If capital entered the production function, research productivity would have to increase even more quickly in income for increases in population to reduce technological change.[3]

If preferences for children and policies for encouraging or discouraging fertility vary among countries, then $\bar{y}$, the level of income that generates zero population growth, will vary as well. If $\delta > 1/(1 - \alpha)$, countries with more pro-natal policies, and hence lower $\bar{y}$, would have lower total research output. Thus, the impact of pro-natal policies on total research output and the cross-section relationship between population and total research output are both ambiguous under this model.

However, even if research productivity increases with income, technological change will still increase with population over time. In the model, population growth is not an exogenous event that causes per capita income to fall, but an endogenous response to technological improvement. Hence per capita income and research productivity remain constant over time as population increases. Over a long time series, therefore, with each person's research

3. If capital enters the production function and the marginal product of capital is set equal to the discount rate, technological change decreases in response to an exogenous increase in population only if $\delta > (\alpha + \gamma)/\gamma$, where α is the share of labor and γ the share of land. Thus, if α were 0.6, γ were 0.1, and the share of capital were 0.3, exogenous increases in population would only reduce total research output if each person's chance of inventing something increased faster than the seventh power of income.

productivity held constant, the speed of technological change will be proportional to total population.[4] Per capita research productivity varies with economic and political institutions, and in cross-section, or over short time series, these fluctuations may be the primary determinants of variation in research output. As long as they are independent of population, however, there will be a positive long-run association between population and research output.

B. Research Productivity as a Function of Technological Level

Jones [1992] proposes a further generalization of the research equation that allows the existing level of technology to affect research output nonlinearly:

$$\dot{A} = gpA^{\phi}. \tag{6}$$

He argues that the assumption $\phi = 1$ is arbitrary, and that since it implies the growth rate of technology will be proportional to the level of population, it is inconsistent with constant or declining rates of TFP growth over the postwar period.[5] Jones argues that $\phi < 1$ is more plausible. In this case, although the absolute increase in A will be proportional to the level of population, the steady state growth *rate* of technology will be proportional to the growth rate of population. To see why, note that

$$\dot{A}/A = gp/A^{1-\phi}. \tag{7}$$

With $\phi < 1$ and constant population, A increases over time, but the ratio $\dot{A}/A$ declines. $\dot{A}/A$ can be constant only if the right-hand side of (7) is constant; that is, if the growth rate of $A^{1-\phi}$ equals the growth rate of p, which implies $(1 - \phi)\,\dot{A}/A = \dot{p}/p$. Thus, given constant population growth at rate n, the steady state growth rate of technology is $\dot{A}/A = n/(1 - \phi)$. Since population growth rates did not increase over the postwar period, and even declined a bit, Jones's model is consistent with constant growth rates of TFP, and may even help explain the productivity slowdown.

4. If population did not adjust instantaneously to income, over short time periods there might be an insignificant, or even negative correlation between population and technological change since fluctuations in $p/\bar{p}$, and thus in income and research productivity, might be significant relative to variation in $\bar{p}$.

5. However, it is possible that $\phi = 1$, since there is evidence of a positive long-run trend in economic growth rates [Romer, 1986], and the stability of TFP growth during the postwar period may reflect temporary idiosyncratic factors, conceptual problems in measuring technological change, or the replacement of nonrival invention as the key constraint on growth by other, rival factors.

Note that the model's predictions for population growth do not substantially change under Jones's more general research equation. Substituting his research equation, (7), into the population growth equation, (4),

$$\frac{\dot{p}}{p} = \frac{1}{1-\alpha} g p A^{\phi-1}. \tag{8}$$

Using $y = \bar{y} = Ap^{\alpha-1}$ to substitute for A,

$$\frac{\dot{p}}{p} = \frac{1}{1-\alpha} g p^{1-(1-\alpha)(1-\phi)} \bar{y}^{\phi-1}. \tag{9}$$

Thus, to take an extreme example, if $\dot{A} = gp$ so that each invention represents a constant absolute increment to the level of technology rather than a constant proportional increment, the growth rate of population will be proportional to p^{α}, approximately $p^{2/3}$, rather than to p. If capital is included in the production function, and if the marginal product of capital equals the discount rate, the growth rate of population is proportional to $p^{1-\gamma(1-\phi)}$, where γ is the share of land. Thus, if γ were 0.1, population growth would be proportional to $p^{0.9}$. Thus, this more general research equation is consistent with both modern and historical data.

C. Research Productivity as a Function of Population

I have so far assumed that each person's research productivity is independent of population. However, this research equation can be further generalized to allow each person's research productivity to depend on the size of the population. Citing the concentration of innovation in cities, Kuznets [1960] argues that research productivity per capita increases with population since higher population allows more intensive intellectual contact and greater specialization. Even without these effects, both Aghion and Howitt [1992] and Grossman and Helpman [1991] find that total research output increases faster than proportionally with population due to increases in the size of the market. On the other hand, higher population might decrease research productivity by increasing duplication of effort. The general formulation $\dot{A} = gp^{\Psi}A^{\phi}$ encompasses both possibilities.

Jones shows that this formulation accommodates a wide range of beliefs about the determinants of research output. Since $y = Ap^{\alpha-1}$, any research equation in which $g = ky^{\delta}$ can be represented in this form. Similarly, the assumption that $\dot{A} = kY$, as in Barro and Sala-i-Martin [1992], which might hold if people invest a constant fraction of their income in a constant returns research sector, can be represented in this framework as $\dot{A} = kAP^{\alpha}$. As Jones demonstrates, under this research equation, exogenous population growth at rate n generates steady state technological change of $\dot{A}/A = \Psi n/(1 - \phi)$. Combining this research equation with the Malthusian population determination equation, and substituting for A using $y = Ap^{\alpha-1}$ yields a growth rate of population proportional to $p^{\Psi-(1-\alpha)(1-\phi)}$.

Under this more general research equation, the finding that population growth rates are roughly proportional to population does not, by itself, separately identify Ψ and ϕ, the exponents on P and A. This suggests an alternative model consistent with the rough proportionality between population and its growth rate over most of history. If $\dot{A} = A^{(2-\alpha)/(1-\alpha)}$, so $\Psi = 0$, and $\phi = (2 - \alpha)/(1 - \alpha)$, which is approximately four for $\alpha = 2/3$, population would have no effect on technology, but the growth rate of technology would increase exogenously at a speed that caused the growth rate of population to be proportional to its level. However, it is possible to rule out this alternative model. If $\phi = 4$, a doubling of A, such has occurred in the postwar period, would cause an eightfold increase in the growth rate of technology. In fact, it is possible to rule out any model with $\phi > 1$, since the change in the growth rate of technology over time in such a model, even in the case of no population growth, is

$$\text{(10)} \qquad \frac{\partial}{\partial t}\left(\frac{\dot{A}}{A}\right) = (\phi - 1)(gp^{\Psi}A^{\phi-1})^2.$$

Thus, if $\phi > 1$, not only is the growth rate constantly increasing, but it is increasing at a faster and faster pace, since A is increasing. This has not been the case empirically: growth rates of per capita income increased from 0.5 percent per year to 2 percent per year over the course of the nineteenth century, and they certainly have not increased by more than another 1.5 percent per year to more than 3.5 percent per year over the twentieth century. Hence ϕ must be less than or equal to one. While time series evidence cannot exclude the possibility that $\dot{A}$ is a complicated function of A such that ϕ was approximately four until recently, but is now less than one, this seems both less parsimonious and less plausible than a model in which research requires human activity. Moreover, such a model would require a decrease in the extent to which one innovation makes another more likely, which seems dubious, given the increased role of systematic science relative to tinkering in modern technological progress. Section IV provides cross-section evidence against models in which technological change is independent of population by showing that among societies without technological contact, those with larger population had faster technological change.

Under this generalized model, $\dot{p}/p$ is proportional to $p^{\Psi-(1-\phi)(1-\alpha)}$. Since $\dot{p}/p$ has historically been roughly proportional to p, $\Psi - (1-\phi)(1-\alpha)$ must be roughly equal to one. Since $\phi \leq 1$, this implies that Ψ must be approximately equal to, or greater than, one. Thus, the speed of technological change must increase at least in rough proportion to population.

It is possible to fully identify ϕ and Ψ if one takes Jones's steady-state equation under exogenous population growth, $\dot{A}/A = \Psi n/(1-\phi)$, as characterizing the modern period, and combines it with the historical evidence that $\Psi - (1-\alpha)(1-\phi) \approx 1$ under Malthusian population determination.[6] Assuming that TFP growth is 2 percent a year, population growth in the high g economies is 1 percent a year, and $\alpha = 2/3$, these two equations imply that $\phi \approx 2/5$ and $\Psi \approx 6/5$.

To summarize, a generalized version of the research equation is consistent with low research productivity in some populous countries, with the possibility that exogenous increases in population reduce research productivity, and with constant growth rates of technology in recent history. Moreover, a model combining this generalized research equation with the Malthusian population determination equation of Section I generates predictions for the growth of population over time that are qualitatively similar to those of the simple model of Section I, and thus match most of the history of population growth.

III. Population as a Function of Technology

This section generalizes the Malthusian population determination equation of Section I and combines it with the generalized research equation in a full model. The simplified model of Section I assumed that population adjusted instantaneously to its steady state. This section shows that if population grows at finite speed when income is above its steady state, per capita income will rise over time. If population growth declines in income at high levels of income, as is consistent with a variety of theoretical models and

6. I thank Robert Lucas for suggesting this.

with the empirical evidence, this gradual increase in income will eventually lead to a decline in population growth.

Section I's assumption that population growth rates were infinite above the steady state level of income made the model tractable, but it is unrealistic. The full model makes the more plausible assumptions that population growth is a continuous function of income, $n(y)$; that at zero income, population growth is negative due to high mortality; and that at some level of income, population growth is positive, since the human race would have died out otherwise. Under these assumptions, there will be some stable steady state level of income, $\bar{y}$, such that $n(\bar{y}) = 0$, and $n'(\bar{y}) > 0$. $\bar{y}$ need not be a physical subsistence level of income, and it could vary between countries, depending on incentives for fertility.

Section I assumed that population growth monotonically increased in income. However, theory suggests that higher levels of income and technology may reduce fertility by increasing wages and thus the value of time [Schultz, 1981], by increasing education [Becker, 1981], by changing the pattern of intergenerational transfers [Willis, 1982], and by increasing the relative value of women's time [Galor and Weil, 1992]. Moreover, Lee's [1987] survey of empirical studies, and studies cited in Becker [1981], indicate that over most of history, at low levels of income, population growth increased with income, but that in recent times, when incomes have been higher, fertility has decreased with income. I shall therefore assume that population growth increases in income at low levels of income and then decreases in income at high levels of income, as depicted in Figure II. This pattern could arise, for example, if raising children entails costs both in goods and time, mortality falls with income, and utility equals $A \ln (K - K^*) + B \ln (c - c^*) + c$, where K is the number of children and c is consumption. When describing the asymptotic behavior of the system, I shall generally assume that $\lim_{y \to \infty} n(y) \geq 0$, although this assumption is not crucial to the analysis over the historical period discussed in this paper.

The simple model of Section I can be considered as an approximation of the full model in which $n'(\bar{y}) = \infty$, so that population adjusts instantaneously to $\bar{p}$, its steady state level. The differential equation for p therefore drops out, and only the differential equation for A remains. This single differential equation approximation will be more accurate when the speed at which population adjusts to income is high relative to the speed of technological change. However, over time the speed of adjustment

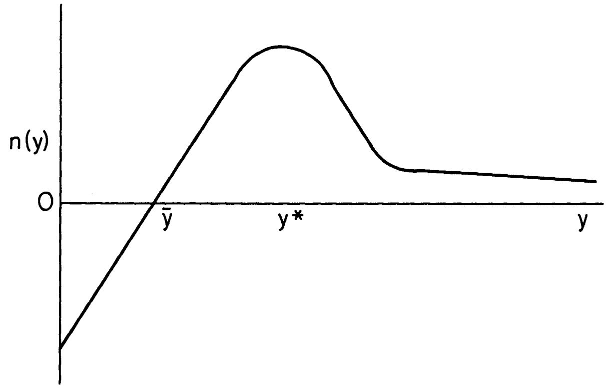

FIGURE II
Population Growth Versus Income

of population to income, which is a constant, declines relative to the rate of technological change, which is constantly increasing.[7] The single differential equation approximation therefore breaks down at high enough levels of population, and it is necessary to examine the full two-differential equation model. This model cannot be solved analytically, but a phase diagram analysis demonstrates that per capita income increases over time, and that eventually this causes growth rates of population to fall.

Before proceeding to the phase diagram analysis, it is worth discussing the intuition for why income must increase over time in the case of the simple research equation of Section I, in which ϕ and Ψ both equal one. Recall that per capita income could be stable only if the growth rate of population equaled $1/(1-\alpha)$ times the growth rate of technology. Given a population $p(0)$ at time 0, this implies that income could be stable only if $\dot{p}/p = gp(0)/(1-\alpha)$. As illustrated in Figure III, to generate population growth at this rate according to the $n(y)$ function, income would have to equal $y(0)$. If income were less than $y(0)$, population growth would lag behind technological change, causing per capita income to grow. Conversely, if income were greater than $y(0)$, population growth would

7. I implicitly assume that $g > 0$. In the absence of technological change, that is, if $g = 0$, the model reduces to a purely Malthusian system, and produces behavior similar to the logistic curve biologists use to describe animal populations facing fixed resources.

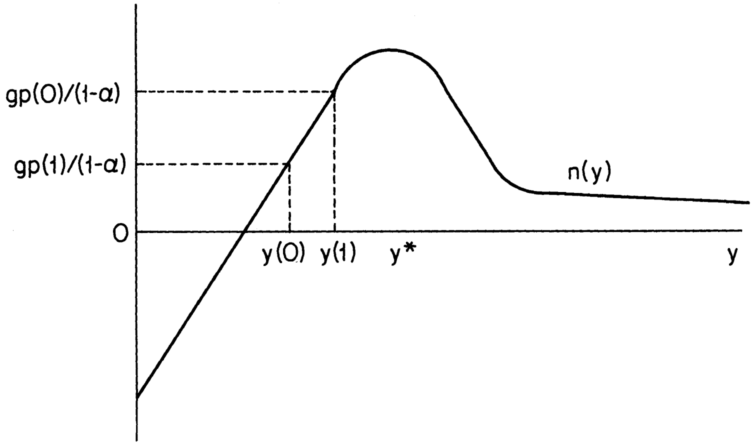

FIGURE III

outstrip technological change, causing per capita income to fall. Now consider the situation at some future time, with population $p(1)$. Income could now be constant only if $\dot{p}/p = gp(1)/(1-\alpha)$. But to generate population growth at this rate, income would have to be $y(1)$. Hence there can be no steady state level of income. Income gradually increases over time with the rate of technological change, which itself increases with population. Once p is large enough that $gp/(1-\alpha) > n(y^*)$, population growth cannot keep up with technological change, the growth rate of per capita income increases, and population growth declines.

The argument above is heuristic and limited to the $\phi = 1$, $\psi = 1$ case, but a phase diagram analysis shows that under the general research production function there will be a period of increasing income and population growth rates, and that eventually income will reach y^*, causing population growth rates to fall. Figure IV shows the phase diagram in population-income space, with one possible configuration of the $\dot{y} = 0$ locus.[8] The $\dot{p} = 0$ locus is the horizontal line along which $y = \bar{y}$. As the arrows indicate, population increases for income greater than $\bar{y}$, and decreases for income less than $\bar{y}$.[9] The $\dot{y} = 0$ locus is given by taking logarithms of the

8. I thank Elhanan Helpman for his great assistance with this phase diagram analysis.
9. If $\lim_{y \to \infty} n(y) < 0$, then there will be another $\dot{p} = 0$ line at high income.

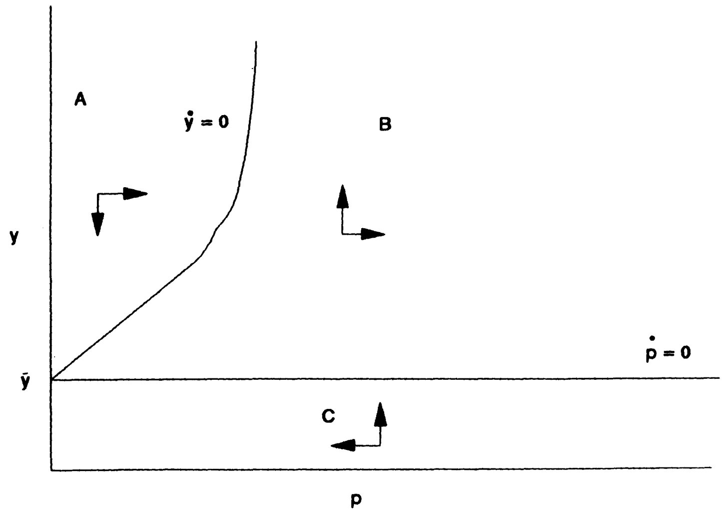

FIGURE IV
Phase Diagram in Population-Income Space

equation $y = Ap^{\alpha-1}$, and differentiating with respect to time:

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} + (\alpha - 1)\frac{\dot{p}}{p}. \tag{11}$$

Income is constant when the growth rate of technology equals $(1 - \alpha)$ times the growth rate of population. Substituting for each of these growth rates, the $\dot{y} = 0$ locus is

$$\dot{y}/y = gp^{\Psi}A^{\phi-1} + (\alpha - 1)n(y) = 0, \tag{12}$$

and since $A = yp^{1-\alpha}$, this can be rewritten as

$$\dot{y}/y = gp^{\Psi-(1-\phi)(1-\alpha)}y^{\phi-1} + (\alpha - 1)n(y) = 0. \tag{13}$$

As noted earlier, I assume that $\phi \leq 1$ and that $\Psi - (1 - \phi)(1 - \alpha) > 0$, which is a weak condition since α is close to one.

To find the shape of the $\dot{y} = 0$ locus, note that it must contain the point $p = 0$, $y = \bar{y}$. Since $\dot{y}/y$ increases in p, but can either increase or decrease in y, there can only be one level of p on the $\dot{y} = 0$ locus corresponding to a given level of y, but there may be multiple levels of y on the locus corresponding to a given level of p. The $\dot{y} = 0$ locus must lie above the $y = \bar{y}$ line for all $p > 0$, because on that line technological change is positive and population growth is zero, so income is increasing. To the right of the $\dot{y} = 0$ locus, the growth rate of technology is high relative to the growth rate of population, so $\dot{y}$ is positive. Correspondingly, to the left of the $\dot{y} = 0$ locus, $\dot{y}$ is negative.

No matter where the economy starts, it winds up in region B. If it starts in region A, with high income relative to population, population increases quickly relative to technology. Per capita income therefore falls until the $\dot{y} = 0$ locus is crossed and the trajectory enters region B. If the economy starts in region C, below the $\dot{p} = 0$ locus, with low income relative to population, population declines, and per capita income rises until the trajectory crosses the $\dot{p} = 0$ locus and enters region B. (It is impossible for any trajectory to cross the axis representing zero population, since the slope of a trajectory is

$$\frac{dy}{dp} = \frac{\dot{y}}{\dot{p}} = \frac{y[gp^{\Psi-(1-\phi)(1-\alpha)}y^{\phi-1} - (1-\alpha)n(y)]}{pn(y)}. \tag{14}$$

Hence for $y < y^*$, as p approaches zero, the trajectory becomes vertical, crosses the $\dot{p} = 0$ axis, and enters region B.)

Once the trajectory is in region B, it remains there, with population and income both increasing indefinitely. Income must eventually reach y^*, the level above which population growth slows. To see why, note that since $\dot{y} > 0$, if y is not asymptotically constant, it must eventually attain a level greater than y^*. On the other hand, y cannot asymptote to a constant level which induces positive population growth, since this would lead to a positive steady state growth rate of technology and income, contradicting the original assumption of asymptotically constant income. Hence if per capita income asymptotes to a constant, it must be to a level that generates zero population growth, and is therefore greater than y^*.

As income rises above y^*, population growth rates decline. If $\phi = 1$, so that the level of technology enters the research production function linearly, growth rates of technology continue to increase because population continues to increase. If $\phi < 1$, so that the level of technology enters the research production function less than linearly, growth rates of technology are likely to continue to increase for some period after $y = y^*$, because of the delayed effects of the prior increases in population growth rates. The growth rate of A depends on the initial values of A and y and on a weighted sum of past population growth rates, and as y increases above y^*, this sum is increasing. Decreasing growth rates of population and increasing growth rates of technology lead to an increase in the growth rate of income per capita.

The asymptotic behavior of population depends on ϕ and on $\lim_{y\to\infty} n(y)$. If $\lim_{y\to\infty} n(y) < 0$, then population asymptotically approaches zero for any ϕ. If $\phi = 1$ and if $n(y)$ goes to zero sufficiently quickly as income increases, population is asymptotically constant. Since the growth rate of technology is proportional to the level of population, the growth rate of technology also asymptotes to a constant. On the other hand, if $\phi = 1$ and $\lim_{y\to\infty} n(y) > 0$, then both population and the growth rate of income increase without bound.

If $\phi < 1$, as in Jones's research equation, the steady state growth rate of technology is $\Psi n/(1 - \phi)$, given constant population growth at rate n. Since $y = Ap^{\alpha-1}$, the steady state growth rate of per capita income will be

$$\frac{\dot{y}}{y} = \left[\frac{\Psi}{1-\phi} + (\alpha - 1)\right] n. \tag{15}$$

In summary, if population adjusts to income at finite speed, then income will gradually rise over time as the growth rate of technology increases. If, in addition, population growth declines with income at high levels of income, there will eventually be a demographic transition, and, for plausible parameter values, steady state growth rates of population, technology, and income. A generalized version of the model is thus at least qualitatively consistent with the recent, as well as long-run, history of population.

IV. Empirical Tests

This section tests the model with both time-series and cross-section population data. The first subsection tests the model's prediction that population growth rates will be roughly proportional to population levels over most of history, using an approach similar to that of Von Foerster, Mora, and Amiot [1960]. They do not build an explicit economic model (they were electrical engineers, not economists), but simply posit an equation in which population growth increases with population, show that it describes the data well, extrapolate it into the future, and conclude, presumably in jest, that world population will become infinite on Friday, the thirteenth of November, 2026. As noted in the previous section, the generalized model predicts that population growth rates will eventually decline—due not to overpopulation and environmental collapse, but to increased income and declining fertility. The second subsection shows that among societies without technological contact, those with greater land area, and hence greater initial population, had faster technological change, as the model predicts.

A. Testing the Model with Population Data

The single differential equation approximation of the full model in Section I predicts that for most of history a regression of population growth on population will generate an intercept of zero and a coefficient on population of $g/(1 - \alpha)$. More generally, under the Jones research production function, the growth rate of population will be proportional to the level of population raised to the power $\Psi - (1 - \alpha)(1 - \phi)$.

In contrast, under the null hypothesis that population is limited by technological change that is independent of population, there would be no correlation between population levels and subsequent growth rates, so the coefficient on population would be zero, and the intercept would be positive. This section tests the model using the data on world population in Table I. Decennial estimates from 1920 on were compiled primarily from United Nations sources. The figures from 10,000 B.C. to 1900 are from McEvedy and Jones [1978]. Their estimates of population after 200 B.C. were obtained by aggregating population estimates for individual geographic regions taken from other authors. These in turn are based primarily on historical sources, such as Roman and Chinese censuses. In contrast, estimates of population prior to 200 B.C., are based on archaeological and anthropological evidence. Population figures before 10,000 B.C. are from Deevey [1960].[10]

Clearly, the population estimates are subject to measurement error, but i.i.d. measurement error or other unmodeled i.i.d. shocks will not only make it harder to pick up any relationship between population growth and levels, but will actually bias the results against the integrated model. This is because undermeasurement of population in period t will cause measured growth from period t to period $t + 1$ to be greater than actual growth, so that it will appear that low levels of population cause high growth rates.

A more serious problem would be systematic bias due to an implicit model in the minds of those who constructed the data.

10. The main data set starts with *homo erectus,* one million years ago, since he invented tools, and therefore should be subject to the model.

TABLE II
POPULATION GROWTH AS A FUNCTION OF POPULATION[a]

Dependent variable: GRPOP (standard errors in parentheses)					
	(1)	(2)	(3)	(4)	(5)
POP	0.524	0.537	0.504	0.548	1.11
	(0.0258)	(0.0334)	(0.0367)	(0.0377)	(0.155)
CONS	−2.26 E-3	−0.0323	3.79 E-4	−0.0571	−0.190
	(0.0355)	(0.0538)	(0.00115)	(0.0252)	(0.0600)
Sample	Full sample	After −200	Full sample	After −200	Evenly Spaced
Weight	unweighted	unweighted	RTGAP	RTGAP	unweighted
n	37	27	37	27	11
R^2	0.92	0.91	0.62	0.79	0.850
DW	1.10	1.14	0.84	1.52	2.42

a. Population is in billions, and growth rates are in percentages, in this and subsequent tables.

However, to the extent that McEvedy and Jones's discussion reveals any implicit model, it is not one similar to that of this paper, but a Malthusian model in which population increases after major exogenous technological changes, such as the agricultural revolution, and then levels off again until the next round of inventions. If McEvedy and Jones fit any data points by exponential interpolation, that would also work against the integrated model, and in favor of the null hypothesis of constant exponential growth.

The results reported in Table II strongly reject the null hypothesis that the coefficient on population is zero.[11] Moreover, in most specifications the intercept is insignificantly different from zero, providing additional evidence for the model. To be sure that the early data points do not drive the regressions, Table II also reports results for the period after 200 B.C.

Under the model, the residuals should be stationary, and indeed it is possible to reject the possibility of a unit root in the residuals. An Engle-Granger test gives a Dickey-Fuller t-statistic of

11. Appropriate critical values for one-sided tests of the null against the alternative that the coefficient is greater than zero are given by the upper tail of the Dickey-Fuller distribution. Since these critical values are extremely low [Fuller, 1976, p. 373], the null is even more strongly rejected than implied by the already high t-statistics. Under the model, in which the coefficient on population is greater than zero, the regression standard errors are sensitive to the distribution of the underlying errors, but if these are normal, the usual t-statistic can be used to construct confidence intervals [Anderson, 1959]. I thank Jushan Bai, Andrew Bernard, and Lars Hansen for discussions on this issue.

TABLE III
TESTS FOR HETEROSKEDASTICITY

	Dependent variable: squared residuals (standard errors in parentheses)			
	(1) OLS residuals	(2) OLS residuals	(3) Weighted regression residuals	(4) Weighted regression residuals
CONSTANT	2.00 E-05	1.72 E-05	−6.94 E-04	−7.51 E-04
	(0.011)	(0.012)	(0.012)	(0.012)
1/Period length	1.02	1.02	1.07	1.07
	(0.248)	(0.256)	(0.256)	(0.264)
YEAR		−4.89 E-11		−9.98 E-10
		(5.58 E-8)		(5.76 E-8)
n	37	37	37	37
R^2	0.32	0.32	0.33	0.33
DW	1.74	1.74	1.70	1.70

−4.25, compared with a 1 percent MacKinnon critical value of only 4.23.

Given the uneven period lengths, it is necessary to correct for heteroskedasticity. In theory, the variance of average growth should be approximately proportional to the reciprocal of the period length.[12] Table III reports tests for heteroskedasticity, which indicate that the squared residuals are indeed roughly proportional to the reciprocal of the period length. The variable YEAR is insignificant in explaining the squared residuals, so there is little evidence that measurement error is considerably more severe in the early periods. While the proportional error in the early estimates of population and population growth is no doubt large, there can be no doubt that the magnitudes were tiny. The absolute error in the estimate of the population growth rate over the period 300,000–25,000 B.C. is thus probably smaller than that over the period 1600–1650, and it is the absolute, rather than the proportional error which determines the standard error of the regression. Obviously, this weighting is not perfect, but it seems a better option than putting equal weight on all periods. As a final

12. This would be true under the null hypothesis with i.i.d. shocks, but it only holds approximately under the model, since a shock one period affects growth the next.

TABLE IV
POPULATION GROWTH AS A FUNCTION OF POPULATION: OTHER DATA SETS

Dependent variable: GRPOP (standard errors in parentheses)			
	Durand	Deevey	Clark
POP	0.816	0.522	0.497
	(0.0617)	(0.0295)	(0.0580)
CONSTANT	−0.194	0.0170	−0.0599
	(0.054)	(0.0193)	(0.0698)
n	5	10	18
R^2	0.98	0.98	0.82
DW	3.28	2.30	2.07

check, Table II reports a regression excluding the data at uneven intervals, leaving ten 200-year periods starting at 200 B.C. and one 190-year period from 1800 to 1990.[13]

Results are similar using other data sets. Deevey [1960], Clark [1977], and Durand [1977] have all published estimates of world population over long historical periods, which are replicated in the Appendix. I use McEvedy and Jones as the principal source since their work is most recent, they have the most data points, and their data points are at regular intervals. However, as Table IV shows, population levels are a significant determinant of growth rates in all three of the other data sets. While the discrepancies between the various estimates indicate the magnitude of measurement error, the results reported in Table IV suggest that the conclusion that population growth increases with population is robust to this measurement error.

This paper uses world population data, since technologies such as the use of fire, the making of iron tools, and the domestication of the dog could diffuse over the long time periods analyzed in this paper. However, McEvedy and Jones also provide regional data,

13. Note the higher coefficient on population in this regression. This is to be expected because the model predicts that the growth rate will increase during the course of the period, and with longer periods, the growth rate increases by more over the period. If population at the end of the period is double what it was at the beginning, the growth rate will be twice as high by then. While the model predicts the instantaneous growth rate of population, the population estimates are at discrete intervals. A previous version of the paper, available from the author, derives predicted population growth over discrete, uneven intervals under a deterministic model. It tests these predictions nonlinearly, and shows that the results are similar to those obtained under OLS. Further complications would arise under an explicitly stochastic model, because the variance of the error term would affect the expected path of population.

TABLE V
POPULATION GROWTH AS A FUNCTION OF POPULATION: EUROPE, CHINA, AND INDIA, 200 B.C. TO 1975

Dependent variable: GRPOP (standard errors in parentheses)

	Europe	China	India
POPULATION	1.55	1.21	4.08
	(0.315)	(0.413)	(0.480)
CONSTANT	0.0796	0.0207	−0.275
	(0.0645)	(0.108)	(0.086)
n	22	22	22
R^2	0.55	0.30	0.78
DW	1.55	1.73	0.63

and as Table V shows, regressions using the smaller geographic regions of Europe, China, and India yield similar results.

The hypothesis of stability of the heteroskedasticity weighted regression over time is consistent with the results of recursive residuals, recursive coefficients, CUSUM, and CUSUM squared tests, as shown in Figures V–VIII. The model predicts that population growth will eventually level off and decline due to increased income, and Figure I appears to suggest a break before the last two observations, but a Chow test finds little evidence for a break at 1970. (Periods are referred to by the date at the beginning of the period.) It is possible to find evidence for a break in an unweighted regression,[14] and despite the weakness of the econometric evidence for a break, there is reason to think that the leveling off of population growth in recent decades differs in nature, if not magnitude, from the random variation the world has experienced throughout history. Population growth in recent years has been below the trend line not because of negative shocks from wars, epidemics, or tyranny, but because of increased income.

The low Durbin-Watson statistics may be due to a break in 200 B.C. Given a break in 1970, an additional break at 200 B.C. raises the Durbin-Watson statistic to 1.73 over the period −200 to 1960. A Chow test on the heteroskedasticity weighted regression provides

14. The unweighted recursive residuals stay within or close to the two standard error band until 1960, and then move outside the band, indicating a break, and a Chow test indicates also indicates a break there. The CUSUM test is consistent with parameter stability over the entire period. The CUSUM of squares test moves outside the bands, but this may reflect its sensitivity to heteroskedasticity rather than shifts in the parameters.

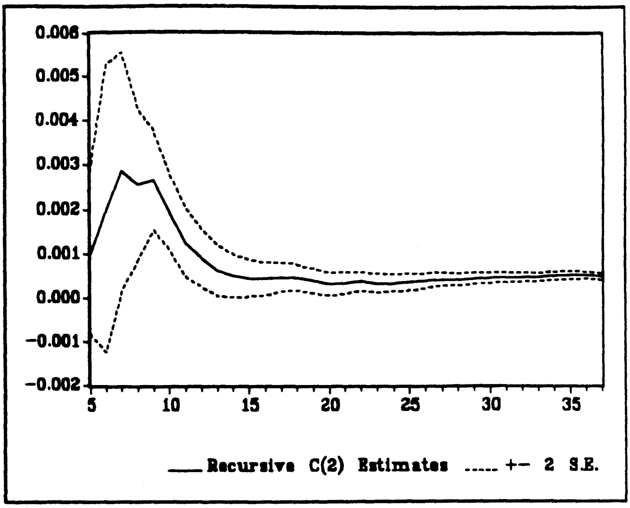

FIGURE V

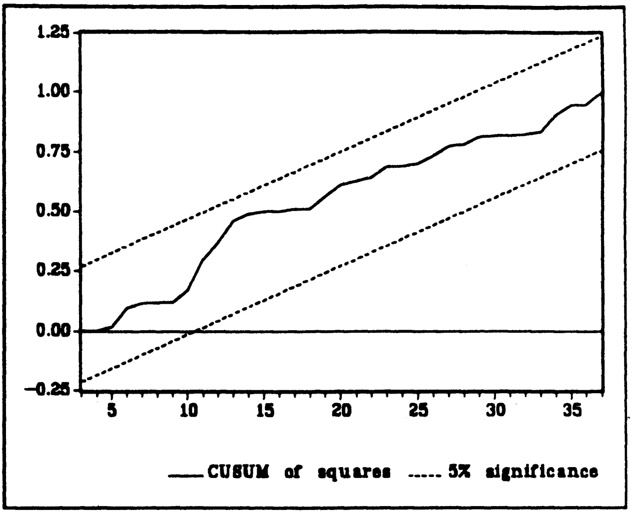

FIGURE VI

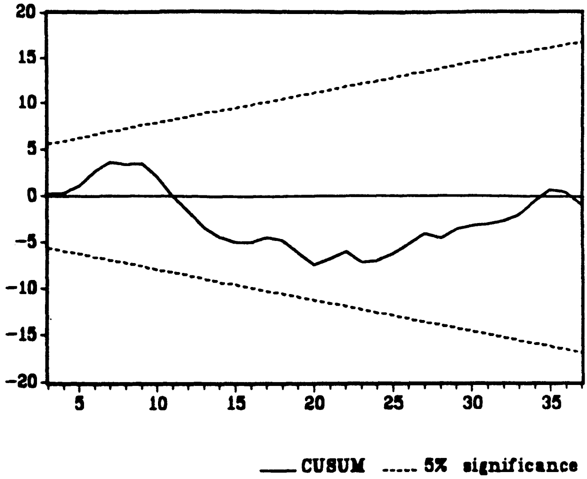

FIGURE VII

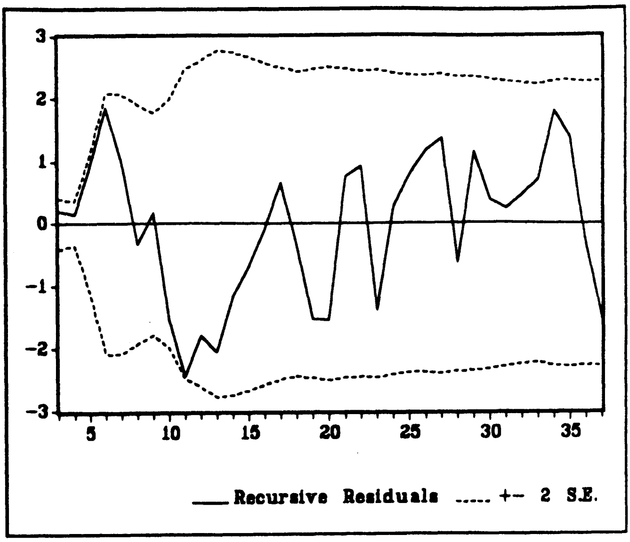

FIGURE VIII

no evidence of a break, but a test on the unweighted regression suggests a higher intercept before 200 B.C. Perhaps this could be attributed to unmodeled population growth in early history due to the settlement of new land[15] and to biological evolution. Chow tests run without a preselected break point will be biased toward rejecting stability, so it is also possible that the apparent break is due to chance.

A Chow test for a break at the industrial revolution in 1800 with the full sample does not reject stability, but if the sample is cut in 1960, it is possible to find a break in 1800. It seems plausible that there was an increase in research productivity due either to socioeconomic factors that increased g, research productivity per capita, or to technological factors that led a group of related inventions to be discovered together, creating a region of the research function with high ϕ.

If the regression is not corrected for heteroskedasticity, it is possible to find periods in which the significant positive relationship between the level of population and its growth rate breaks down, but I do not think that this is too serious a problem with the model. If one considers unweighted regressions over successively lengthier samples, and uses standard t-statistics for a one-sided test, population becomes significant by 4000 B.C. and remains so until the Roman empire begins to decline in the second century. Population is significant again in 1000 and 1100. It becomes insignificant for three periods due to the negative outliers of the Black Death, which reduced Europe's population by a third, and the Mongol conquests, which reduced China's population from 115 million in 1200 to 86 million in 1300. Population becomes significant again before the impact of the industrial revolution on world population. It is significant at all times after 1500, except for the period 1600–1650 with the simultaneous disasters of the Thirty Years' War, which devastated Central Europe, and the fall of the Ming dynasty, which reduced China's population from 160 million in 1600 to 140 million in 1650. If one uses the theoretically more appropriate upper-tail Dickey-Fuller critical values, population is significant through all these negative shocks.[16]

Given the noisiness of the data and the small number of data points, it is unsurprising that by searching over various subsamples it is possible to find periods over which the coefficient is not

15. I am grateful to Abhijit Banerjee and Andrew Newman for this suggestion.

16. This is based on the critical values in Fuller [1976] for a sample size of 25. For more precise estimates of the critical values for smaller samples, Monte Carlo estimates would be necessary.

TABLE VI
ESTIMATES OF $\Psi - (1 - \alpha)(1 - \phi)$

GRPOP = CONST + K*$POP^{\Psi-(1-\alpha)(1-\phi)}$
(standard errors in parentheses)

	(1)	(2)	(3)	(4)
CONST	4.51 E-4	6.25 E-4	−0.038	−0.036
	(0.00117)	(0.0011)	(0.031)	(0.052)
K	0.493	0.507	1.18 E-9	2.13 E-6
	(4.45 E-2)	(4.74 E-2)	(2.13 E-9)	(3.12 E-6)
$\Psi - (1 - \alpha)(1 - \phi)$	1.03	1.22	1.43	0.907
	(0.081)	(0.112)	(0.122)	(0.0965)
Weighting	RTGAP	RTGAP	unweighted	unweighted
Sample	−1,000,000 to 1980	−1,000,000 to 1960	−1,000,000 to 1980	−1,000,000 to 1960
DW	0.859	0.893	1.083	1.537
R^2	0.622	0.578	0.924	0.949
n	37	35	37	35

significant in some specifications. Since this regression is not corrected for heteroskedasticity, it is driven by fluctuations at the end of the sample. With a heteroskedasticity weighted regression, population is significant at all times after 4000 B.C. even using standard t-statistics. When one looks at long periods in which fluctuations average out, there is clear evidence of a secular long-run trend. Population growth was less than 0.00073 percent a year from 200,000 B.C.[17] to 10,000 B.C.; 0.037 percent a year from 10,000 B.C. to the year 1; and 0.073 percent a year from the year 1 to 1600. So I do not think that the positive trend holds only after the industrial revolution. As noted earlier, the recursive residual, CUSUM, and CUSUM squared tests are consistent with stability of the relationship over the entire period. Even using standard t-statistics, population is significant in India by 1600 and in China by 1750, before the impact of the industrial revolution on their populations. Finally, the cross-section evidence in subsection IV.B on regions that had no technological contact before 1500 indicates that the model applied before that date.

Table VI reports results from using nonlinear least squares to estimate the model with the more general Jones research equation,

17. Starting the sample with homo sapiens, 200,000 years ago, works against the model by producing a more rapid early growth rate. I use Deevey's population estimate for 300,000 years ago, which also works against the model.

so that population growth is proportional to $p^{\Psi-(1-\alpha)(1-\phi)}$. Since the last two observations may reflect the demographic transition, it also reports regressions using data up to 1960. While the likelihood function is fairly flat, so these estimates should be taken with a grain of salt, they suggest that $\Psi - (1 - \alpha)(1 - \phi)$ is greater than, or approximately equal to, one. Since ϕ, the degree to which research output increases in the level of technology, cannot be greater than one, Ψ is greater than, or approximately equal to, one.

High R^2's cannot be obtained with any increasing right-hand side variable, since population and its growth rate are not merely increasing variables, but variables that increase at an ever increasing rate. The year, for example, is almost insignificant as a right-hand side variable. Exp (year/k) can drive out population for some values of the constant k, but few obvious economic variables grew exponentially during this period. It is unlikely that per capita income would have much explanatory power, since its growth is unlikely to have matched that of population, which, for example, grew thirty-fivefold from 10,000 B.C. to 200 B.C.

Perhaps it would be possible to explain the data through some other variable, or through a series of particular historical events that caused the growth rate of technology to increase at some periods and decrease at others, without including an effect of population on technology. However, given that a simple model, based on the economic theory of technology as a nonrival good, is consistent with the data over such a long period, it is not clear why one would want to abandon it for an alternative explanation of the data.

B. Cross-Section Evidence from Technologically Separate Regions

The model implies that if there were no technological contact between regions that started with similar technology and with population proportional to their land area, those regions with greater land area, and hence larger initial populations, would experience faster technological change. Hence they would attain higher levels of technology and greater population densities. To see why, integrate the population determination equation, $dp/p^2 = g\, dt/(1 - \alpha)$, to obtain population at time t in region i, as a function of initial population, p_{i0}:[18]

$$(16) \qquad p_i(t) = \frac{1}{(1/p_{i0}) - (gt/(1 - \alpha))} \qquad t < \frac{1 - \alpha}{gp_{i0}}.$$

18. Note that this would generate infinite population in finite time if it were not for the demographic transition discussed earlier. I thank Serge Marquie and Alan Taylor for assistance with these calculations.

(By Jensen's inequality, the expected value of $p(t)$ for any value of $g/(1 - \alpha)$ would be larger in a model with shocks, since $p(t)$ is a convex function of $p(0)$.) Dividing by land area, T_i gives

$$(17) \qquad d_{i,t} = \frac{1}{(1/d_0) - (gtT_i/(1 - \alpha))} \qquad t < \frac{1 - \alpha}{d_0 gT},$$

where d_{it} denotes the population density of region i at time t and d_0 denotes the initial common population density. It is straightforward to write an equivalent expression for A_{it}, the level of technology, as a function of land area, since d_{it} is proportional to $A_{it}^{1/1-\alpha}$. The model also predicts that the elasticity of density with respect to land area will be $(1 - \alpha)d_0 gtT_i/(1 - \alpha - d_0 gtT_i)$ and thus will increase with land area, T_i. In contrast, under an alternative model of exogenously increasing growth rates of technology, independent of population, there would be no correlation between land area and levels of technology and population density.

The melting of the polar ice caps at the end of the ice age, around 10,000 B.C., and the consequent flooding of land bridges, provide a natural experiment that nearly eliminated contact between the old world, the Americas, mainland Australia, Tasmania, and Flinders Island.[19] As the model predicts, in 1500, just after Columbus' voyage reestablished technological contact, the region with the greatest land area, the Old World, had the highest technological level. The Americas followed, with the agriculture, cities, and elaborate calendars of the Aztec and Mayan civilizations. Mainland Australia was third, with a population of hunters and gatherers. Tasmania, an island slightly smaller than Ireland, lacked even such mainland Australian technologies as the boomerang, fire-making, the spear-thrower, polished stone tools, stone tools with handles, and bone tools, such as needles [Diamond, 1993].[20] Flinders Island, near Tasmania, has only about 680 square kilometers of land, and according to radiocarbon evidence, its last inhabitants died out about 4000 years after they were cut off by the rising seas—suggesting possible technological regress.[21] If techno-

19. Different land bridges were flooded at different dates. Flinders Island was probably cut off only 8700 years ago.

20. Diamond [1993] explicitly attributes Tasmania's low technological level to its low population.

21. The Tasmanians' technological stock actually depreciated: they lost the ability to make bone tools, for example, which archaeological evidence shows they once possessed. On the other hand, they probably invented a crude boat about 4000 years ago. Introducing depreciation of technology into the model could create zero or negative technological change if population or income, and hence research productivity, were low enough. This creates a richer model with multiple steady states and paths to extinction. While these might be relevant for some particular cases, such as Flinders Island, I believe they are of limited importance when looking at the world as a whole.

TABLE VII
POPULATION AND POPULATION DENSITY, C. 1500

	Land area (million km^2)	Population c. 1500 (millions)	Population/(km^2)
Old World[a]	83.98	407	4.85
Americas[b]	38.43	14	0.36
Australia[c]	7.69	0.2	0.026
Tasmania	0.068	0.0012–0.005	0.018–0.074
Flinders Island	0.0068	0.0	0.0

a. Sub-Saharan Africa is included in the old world, since there was some contact across the Sahara.
b. There are a wide range of population estimates for the Americas and Australia at the time of European arrival, and McEvedy and Jones's are at the low end. However, higher estimates would not affect the rank ordering.
c. Estimates for Tasmania are based on the *Encyclopaedia Brittanica*.

logical change were actually independent of initial population, the chance that technology levels in the four inhabited regions would be ranked in this same order as land area is only 1 in 24. If Flinders Island is included, the chance drops to 1 in 120.

Although their isolation was never as complete as that of the regions discussed above, ancient Britain and Japan also fit the model. When the land bridge between ancient Britain and Europe was cut off, around 5500 B.C., Britain fell technologically behind Europe.[22] Agriculture was introduced around 4000 B.C. by neolithic immigrants from Europe and metallurgy was brought by immigrants from the low countries around 2300 B.C. Ancient Japan was settled by paleolithic people from the mainland before its land connections to Asia were cut off by rising seas. Although its prehistory is murky, Japan's paleolithic people seem to have been very primitive: they lived in pits or caves rather than building even primitive structures, and no bone or horn artifacts associated with neolithic people in the rest of the world have been found in Japan. Immigrants from Asia bearing culture from Korea and China later brought more advanced technology to Japan.

Table VII shows that estimated population density in 1500 increases with land area, as the model predicts. Tasmania's raw population density appears similar to that of mainland Australia, but its population per unit of quality adjusted land is probably lower, since more than half of Australia is inhospitable desert, receiving less than 30 centimeters of rainfall a year, while most of Tasmania has relatively favorable conditions.

22. Information on ancient Britain and Japan is from *Encyclopaedia Britannica* [1987].

Using equation (17), it is possible to make quantitative predictions of each region's density in 1500, given some heroic assumptions. Assuming that technological contact was cut off in 10,000 B.C., so $t = 11{,}500$, that d_0, initial density for all regions, was equal to McEvedy and Jones's estimated world population density of 0.030729 per square kilometer, and that the quality of land in all four areas was the same, so T_i corresponds to the entries in Table VII, then in order to generate a population density of 4.85 per square kilometer in the Old World in 1500, $g/(1 - \alpha)$ would have to equal 0.0335 per billion people, and this would have produced population densities of 0.0308 per square kilometer in Tasmania, 0.0338 in mainland Australia, and 0.0564 in the Americas. The model's prediction that a given percentage discrepancy in land area between two regions will have more of an effect on population density at high levels of land area matches the data: mainland Australia's population density is of the same magnitude as Tasmania's despite having more than 100 times the land area, while the old world, with eleven times Australia's land area, has more than 150 times its density. Moreover, the model correctly predicts that population densities in Tasmania and Australia would not increase appreciably over the initial density, d_0.[23] However, the model underpredicts population in the Americas relative to that in the Old World, and it requires a higher level of $g/(1 - \alpha)$ than suggested by the regressions of subsection IV.A. These discrepancies may be due in part to underestimation of population in 10,000 B.C.; to inclusion of sub-Saharan Africa in the Old World, despite the extremely limited technological contact across the Sahara; and to differences in land quality or date of technological separation.[24] However, they may also reflect problems with the simple $\phi = 1$, $\psi = 1$, research equation and the model's assumption of instantaneous

23. The calibration assumes that $d_0 = 0.307$, but since the actual population density of Australia was less than this in 1500, it seems likely that Australia had a lower initial density, perhaps due to lower land quality, or due to becoming technologically separate earlier than 10,000 B.C.

24. If population in 10,000 B.C. were 10 million, as some have estimated, and if sub-Saharan Africa were treated as a separate unit from the rest of the old world, the predicted population density in the Americas in 1500 given that in the old world would have been about 0.2 per square kilometer. America's discovery of agriculture may represent a group of related inventions with high ϕ.

The lower value of $g/(1 - \alpha)$ suggested by the time series regressions is due in part to the assumption that technology could diffuse across regions. If initial population were 10 million and if the world were taken as a unit, the estimated value of $g/(1 - \alpha)$ would be 0.00849. Moreover, since equation (17) does not allow for a stochastic term, it will generate a higher estimate of $g/(1 - \alpha)$ than a time series regression.

technological diffusion within regions.[25] Given the strong assumptions required for calibration, the low quality of the data, and the model's sensitivity to initial conditions, it is surprising that so crude a model matches the data this well.

In sum, regions with greater land area, and hence greater initial population, attained higher technological levels and population densities, as the model predicts. While we cannot precisely determine the nonlinear function relating initial population to final technological level and population density, the data are difficult to reconcile with models in which technological change is independent of population.

V. Conclusion: Implications for Policy and Theory

Following Lee [1988], this paper constructs an integrated model of population growth and technological change. It assumes that each person's chance of inventing something is independent of population, so that total research output increases in proportion to population. Over the historical period when population was limited by the available technology, the model therefore predicts that the growth rate of population will be approximately proportional to the level of population. Per capita income gradually increases with the growth rate of technology, and eventually this causes population growth to slow. Empirical evidence supports the model: through most of history the growth rate of world population has been approximately proportional to the level of population. Moreover, among societies with no opportunity for technological contact, those with greater initial population attained higher technology levels and population densities. These facts are difficult to reconcile with prevailing growth models in which technological change is independent of population.

The model of continuous acceleration of population and technology proposed here can be contrasted with models involving discontinuous breaks, such as multiple equilibria models in which the economy either stagnates or experiences steady state growth. These models typically make few predictions about when the economy will be in each equilibrium. Moreover, their focus on technological stagnation as the alternative to steady state growth reduces most of history to the category of stagnation, despite such inventions as the wheel, Euclidean geometry, the plow, and the compass. It is ironic that growth theorists are building models with

25. David Romer has pointed out that a model with $\phi > 1$ and $\Psi < 1$ could match the population of the Americas, although it would be inconsistent with data from the modern period.

sharp breaks at a time when most development economists reject the notion of takeoff and many economic historians stress continuity rather than a discontinuous industrial revolution.

Future research may seek to quantitatively model the demographic transition and to allow for slow diffusion of technology and for stochastic shocks to population and technology. This paper has abstracted from fluctuations in research productivity per capita, since it focuses on extremely long periods over which they may average out. However, the study of how economic and political institutions affect research productivity remains critical for understanding time series dynamics over shorter periods and cross-section differences between countries, since in these contexts the variance of research productivity per capita is often large relative to that of population.

Although the model is designed to reflect historical, rather than current conditions, it is worth considering its implications for the present, both for policy and for growth theory. If research productivity per person depends on income, the short-run impact of pro-natal policies, such as tax allowances for children, on the speed of technological change is ambiguous. For example, child subsidies that increase birth rates might lower research productivity per capita. However, in the long run this model implies that faster population growth leads to faster technological change. For $\phi = 1$, the growth rate of technology equals research productivity per capita times population, and the one-time fall in research productivity per capita caused by an increase in fertility will eventually be outweighed by the cumulative effect of population growth. For $\phi < 1$, Jones shows that the asymptotic growth rate of technology is proportional to the growth rate of population and is independent of research productivity per capita. However, while pro-natal policies may be growth-enhancing from the point of view of the world as a whole, individual countries may wish to let their citizens choose the privately optimal family size and to free ride off technological innovations made in other countries. Moreover, the model used in this paper does not allow for either exhaustible natural resources or for an ultimate limit on the level of technology, and in models incorporating these features, population growth could reduce long-run income per capita. Thus, the model should not be taken as a call for increased population. It does suggest, however, that economists should conduct further research to measure the growth and welfare effects of population growth under nonrival technology, rather than simply following conventional wisdom and concentrating on the negative effects of population growth.

The model's implications for growth theory are clearer. Most models of endogenous technological change imply that all else equal, higher population spurs technological change. This result, I believe, is due not to any quirk of modeling, but to the fundamental nonrivalry of technology as described by Romer. Perhaps it is possible to argue that technological change is independent of population, and to construct some other explanation of why the growth rate of population has historically been proportional to its level. Perhaps it is even possible to explain why among technologically separate regions, those with higher population have had faster technological change. However, given that our theoretical models of technological change predict that higher population leads to faster technological change, what is noteworthy is not that other models might be able to explain the data, but that an extremely stylized model, based on theory, provides such a good description of the data over such a long period. Endogenous growth theorists have dismissed the population implications of their models as empirically untenable. This paper suggests that we should take them seriously.

APPENDIX

A. POPULATION GROWTH: EUROPE, CHINA, AND INDIA

Year	Europe population (millions)	Europe growth rate (%)	China population (millions)	China growth rate (%)	India population (millions)	India growth rate (%)
−200	26	0.0875	42	0.1157	31	0.0604
1	31	0.0751	53	0.0869	35	0.0795
200	36	−0.0748	63	−0.0864	41	0.0683
400	31	−0.0879	53	−0.0291	47	0.0601
600	26	0.0546	50	0.0000	53	0.0943
800	29	0.1081	50	0.1388	64	0.1053
1000	36	0.2007	66	0.4643	79	0.0494
1100	44	0.2763	105	0.0910	83	0.0355
1200	58	0.3090	115	−0.2906	86	0.0565
1300	79	−0.2751	86	−0.0599	91	0.0639
1400	60	0.3001	81	0.3060	97	0.0792
1500	81	0.2107	110	0.3747	105	0.2513
1600	100	0.0976	160	−0.2671	135	0.2107
1650	105	0.2671	140	0.2671	150	0.1906
1700	120	0.3083	160	0.6819	165	0.1177
1750	140	0.5026	225	0.7660	175	0.1645
1800	180	0.7735	330	0.5525	190	0.3821
1850	265	0.6914	435	−0.1883	230	0.4127
1875	315	0.8543	415	0.5401	255	0.5145
1900	390	0.7463	475	0.4382	290	0.5168
1925	470	0.3657	530	0.4290	330	1.1959
1950	515	0.8378	590	1.3892	445	2.2119
1975	635	—	835	—	775	—

APPENDIX (CONTINUED)

B. DURAND DATA[a]

(Population figures are based on midpoints of Durand's ranges)

Year	Population (millions)	Growth rate (%)
1	300	0.00328
1000	310	0.0996
1500	510	0.1648
1750	770	0.5201
1900	1680	1.1567
1975	4000	—

C. DEEVEY DATA

Year	Population (millions)	Growth rate (%)
−1,000,000	0.125	0.000297
−300,000	1	0.000439
−25,000	3.34	0.0031
−8000	5.32	0.0697
−4000	86.5	0.0108
1	133	0.0835
1650	545	0.2895
1750	728	0.4375
1800	906	0.5750
1900	1610	0.7985
1950	2400	—

D. CLARK DATA

Year	Population (millions)	Growth rate (%)
14	256	−0.00233
350	254	−0.0277
600	237	0.0482
800	261	0.0351
1000	280	0.1579
1200	384	−0.0112
1340	378	0.0762
1500	427	0.1538
1600	498	0.0710
1650	516	0.4338
1700	641	0.2628
1750	731	0.3936
1800	890	0.6282
1900	1668	0.8270
1920	1968	0.8612
1930	2145	0.8701
1940	2340	0.6574
1950	2499	1.6221
1962	3036	—

a. Population figures are based on midpoints of Durand's ranges.

DEPARTMENT OF ECONOMICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY

REFERENCES

Aghion, Philippe, and Peter Howitt, "A Model of Growth Through Creative Destruction," *Econometrica,* LX (1992), 323–52.
Anderson, Theodore W., "On Asymptotic Distributions of Estimates of Parameters of Stochastic Difference Equations," *Annals of Mathematical Statistics,* XXX (1959), 676–87.
Arrow, Kenneth J., "Economic Welfare and the Allocation of Resources for Inventions," in R. R. Nelson, ed., *The Rate and Direction of Inventive Activity* (Princeton, NJ: Princeton University Press for the NBER, 1962).
Barro, Robert, and Xavier Sala-i-Martin, "Economic Growth," manuscript, 1992.
Becker, Gary, *Treatise on the Family* (Cambridge, MA: Harvard University Press, 1981).
Boserup, Ester, *The Conditions of Agricultural Progress* (Chicago, IL: Aldine Publishing Company, 1965).
Clark, Colin, *Population Growth and Land Use* (London: Macmillan, 1977).
Deevey, Edward S., "The Human Population," *Scientific American,* CCIII (1960), 195–204.
Diamond, Jared, "Ten Thousand Years of Solitude," *Discover,* XIV (1993), 49–57.
Durand, John, "Historical Estimates of World Population: An Evaluation," *Population and Development Review,* III (September 1977), 253–96.
Fuller, Wayne, *Introduction to Statistical Time Series* (New York, NY: John Wiley & Sons, 1976).
Galor, Oded, and David Weil, "The Gender Gap, Fertility, and Growth," Working Paper, Brown University, 1992.
Grossman, Gene, and Elhanan Helpman, *Innovation and Growth in the Global Economy* (Cambridge, MA: MIT Press, 1991).
Jones, Charles, "R&D Based Models of Economic Growth," unpublished, Massachusetts Institute of Technology, 1992.
Kuznets, Simon, "Population Change and Aggregate Output," in *Demographic and Economic Change in Developed Countries* (Princeton, NJ: Princeton University Press, 1960).
Lee, Ronald, "Population Dynamics of Humans and Other Animals," *Demography,* XXIV (1987), 443–65.
——, "Induced Population Growth and Induced Technological Progress: Their Interaction in the Accelerating Stage," *Mathematical Population Studies,* I (1988), 265–88.
Malthus, Thomas, *Population: The First Essay* (Ann Arbor, MI: University of Michigan Press, 1978).
McEvedy, Colin, and Richard Jones, *Atlas of World Population History* (New York: Penguin, 1978).
Romer, Paul, "Increasing Returns and Long-Run Growth," *Journal of Political Economy,* XCIV (1986), 1002–37.
——, "Endogenous Technological Change," *Journal of Political Economy,* XCVIII (1990), S71–S103.
Schultz, Paul T., *Economics of Population* (Reading, MA: Addison-Wesley, 1981).
Simon, Julian, *The Economics of Population Growth* (Princeton, NJ: Princeton University Press, 1977).
——, *The Ultimate Resource* (Princeton, NJ: Princeton University Press, 1981).
United Nations, *United Nations Statistical Yearbook* (New York; United Nations, 1952, 1985/6).
Von Foerster, Heinz, Patricia Mora, and Lawrence Amiot, "Doomsday: Friday, 13 November, A.D. 2026," *Science,* IV (1960), 1291–95.
Willis, Robert J., "The Direction of Intergenerational Transfers and Demographic Transition: The Caldwell Hypothesis Reexamined," *Population and Development Review,* Supplement to Vol. VIII, (1982), 207–34.
The World Almanac (New York: Pharos Books, 1991).
Young, Alwyn, "Invention and Bounded Learning by Doing," Working Paper, 1990.